The Philosophical Review, XCIV, No. 2 (April 1985)

BOOK REVIEWS

The Philosophical Review, XCIV No. 2 (April 1985)

INFINITY AND CONTINUITY IN ANCIENT AND MEDIEVAL THOUGHT. Edited by NORMAN KRETZMANN. Ithaca, New York, Cornell University Press, 1982. Pp. 367. $27.50.

This volume collects eleven articles by first-rate scholars on the treatment of the puzzles and paradoxes of infinite divisibility and continuity. The papers are developed from presentations by classicists, historians of philosophy and logic, and historians of science and mathematics at a conference held at Cornell University in 1979. Five of them, by David Furley, Richard Sorabji, Fred Miller, Wilbur Knorr, and Ian Mueller, concern the discussion in antiquity. Furley's piece examines the interpretation of Aristotle's theory of space and time in the Greek commentators. Sorabji shows us how Aristotle's four paradoxes about time in the beginning of *Physics* IV.10 and material from the beginning of *Physics* VI inspired work in late antiquity, particularly certain little-known atomistic theories. Miller's article returns to Aristotle himself, and his reaction to the Atomists of his own day, before taking up the reaction of the later Atomists to Aristotle. Knorr turns our attention to ancient mathematics and its techniques, arguing that mathematics influenced philosophy on the questions at issue more than philosophy influenced mathematics. In Mueller's article the exploration of philosophy's relation to mathematics is continued with an examination of Aristotle's references to attempts at squaring the circle.

The remainder of the book deals with speculations in the fourteenth century. John Murdoch considers Ockham's commentary on the *Physics* in a thorough and scholarly fashion, and fills in the background to the medieval discussion admirably; but he gets confused about Ockham's intentions. Murdoch is corrected on several points in Eleonore Stump's elegant discussion of Ockham's theory of indivisibles in *De sacramento altaris.* The oddly logical connection between Ockham's theory of the Eucharist and his theory of indivisibles is one of the more pleasing sidelights of the medieval discussion of these topics. Edith Sylla discusses theories of qualitative change, in particular those of Walter Burley and Richard Kilvington, contemporaries of Ockham. Calvin Normore relates Burley's notions to modern mathematical work on the continuum, notably that of Dedekind. Norman Kretzmann and Paul Spade deal with two fourteenth-century theories of change, a theory Kretzmann dubs "quasi-Aristotelianism" and the orthodox Aristotelianism of Richard Kilvington. In a very intelligent exchange Kretzmann finds quasi-Aristotelianism seriously deficient, and Spade attempts a partial rehabilitation of the view.

The presentations of this varied group of authors differ widely from one another in scope and intent. Some cast their net wide, forming useful and detailed synopses of whole schools of thought. Others restrict themselves to an intensive analysis of a single text. From previous encounters one expects the philosophers to take a more intensive approach and display greater analytical perspicacity, the classicists to display more widely ranging scholarship, and the historians of science to add to wider scholarship a greater knowledge of mathematics. Part of the point of a project such as this is to bring these complementary abilities together. These expectations were largely met, though one must be amazed at the extent to which Richard Sorabji combines all the virtues .

David Furley, in an otherwise interesting and useful piece, seemed to misconstrue and underestimate Simplicius' interpretation of Aristotle (pp. 20–26), which not only reconciles the *Categories* with the *Physics*, but provides a plausible reading of Aristotle's intention. If one notes that a part of a body *does* occupy a place, as the *Categories* holds, but only occupies a place *considered as a part of the body in question*, not *kath' heauta* but in virtue of the body's being *kath' heauta* in a place, the apparent contradiction can be straightened out. Moreover, Furley misses Aristotle's point in *Physics* V 3. There Aristotle's intention is to allow that even though bodies are not identical with their places, geometrical figures are (209a7 ff., cited by Furley on p. 21). Geometry is the science of places. Thus the extremity of body, *considered as an extremity*, has a place only accidentally, in virtue of the body's place, but *considered in itself*, as a geometrical figure, it is a surface and its place *is* itself. So the extremities of two different bodies can be together, for though they are different considered as extremities of different bodies, they are the same place, considered in themselves. Thus Simplicius' insistence that the two extremities are "in one place, in the sense in which a surface according to its nature is in place" (*in Phys.* 871.8, cited p. 26) is exactly right, and the sense in which they are in one place according to their nature is clear: they are identical to one and the same place.

The articles by Sylla and Normore concern themselves with Dedekind's analysis of continuity embodied in his definition of real numbers as rational cuts. Both fall prey to serious misunderstandings of Dedekind's view and its relation to Aristotle, and I must attempt very briefly to indicate a better view. Sylla's confusion first reveals itself when she remarks that medieval thinkers generally held that points and indivisible degrees cannot be continuous with one another on Aristotle's view. For Aristotle continuity is a pairwise concept, two things being continuous if they share an extremity, so she is right. But she goes on to say that modern mathematics represents a line as an infinite set of points, so that points *can* form a continuum, and so can be continuous. Of course, it does not follow from a continuum being formed by an infinite set of points that any two of them are continuous, as Dedekind knew. Nor is it the case, as Sylla suggests (p. 256) that there is some deep problem with Dedekind's theory that incapacitates it for the work of describing linear continua. One point on a linear continuum can terminate either of two lines, as Aristotle admits, and Dedekind's insistence that *two* cuts correspond to each point simply mirrors this fact. If we must find what each cut *uniquely* corresponds to we must identify a pair of lines, one containing its extremity and the other not. Normore's approach to the same material strikes me as far more confused. To pick the worst mistake, the analyses of Kilvington and Furley in no way imply that there is a moment immediately after a given time having all the strange properties that Normore identifies (p. 264). Those analyses are, indeed, deliberately constructed to avoid the problems he raises, and succeed in doing so. I had the impression that Normore and Sylla were both committed to the view that Dedekind's and Aristotle's views could not be in agreement because Dedekind and Aristotle had different ontological commitments. In fact it seems that the two views are not at odds and do concern the same subject matter. Though they may disagree on whether continua really *really* exist, or whether they are simply collections of points, they do agree that continua really exist, and are interested in continua. If they make different ontological presuppositions, because Aristotle constructs points from spatial continua, and Dedekind spatial continua from points, then perhaps that tells us something odd about ontological commitments. Or maybe they *don't* make different ontological commitments.

Kretzmann's book is very valuable, essential for any student of the topic it deals with, and most useful for those interested in medieval logic and physical science (or natural philosophy) and in the history of mathematics in antiquity and the Middle Ages. Although the articles are not of uniform quality, all but a few are very good indeed, and the book as a whole provides a fascinating, up-to-date discussion of one of the more exciting fields of scholarly archaeology.

JOHN LONGEWAY

University of Wisconsin at Parkside

The Philosophical Review, XCIV No. 2 (April 1985)

THE GREEKS ON PLEASURE. By J. C. B. GOSLING AND C. C. W. TAYLOR. New York, Oxford University Press, 1983. Pp. xiii, 497.